A Theoretical Study of the Cr:BeAl$_2$O$_4$ Laser Passively Q-switched with Cr:YSO Solid State Saturable Absorber

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The theory of passive Q-switching of solid-state lasers with slow-relaxing solid-state saturable absorbers is investigated in this paper. Passive Q-switching performance of the Cr:BeAl$_2$O$_4$ laser with the Cr:YSO solid-state saturable absorber is numerically studied. With typical optical pumping, output coupling, and saturable absorber doping concentration, an energy output of more than 200 mJ in a laser pulse duration of less than 30 ns may be obtained.

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I. Introduction

Giant laser pulses may be generated by means of Q-switching [1, 2]. Fast optical shutters can be made by using the electro-optic effect or the acousto-optic effect in some solid-state crystals. These active Q-switching systems are effective and have been widely used in many of the industrial applications because they operate reliably over an extended period of time and can be triggered at any moment within the pumping cycle. However, the overall laser system is rather complicated. Compared to active Q-switching, passive Q-switching with a saturable absorber is economical and simple because it requires less optical elements inside the laser cavity and no outside driving circuitry. Passive Q-switching is a better choice for those applications where compactness of the laser is a prime requirement.

Organic dyes have long been used as saturable Q-switch absorbers [3, 4]. The dye cell is highly absorptive at low light intensity and can be bleached when the light becomes intense. Short laser pulses with high peak power can be obtained with the use of a dye Q-switch. However, dye Q-switches suffer from poor chemical stability and inadequate thermal properties. To overcome this problem, a flow system has been used at the expense of compactness and simplicity [1].

Semiconductors used as saturable absorber Q-switches have also been reported in the 1-$\mu$m [5], 2-$\mu$m [6], and 3-$\mu$m [7] regions. Semiconductors possess high small-signal absorption coefficients when the incident light has photon energies higher than the bandgap. However, if the wavelength of the incident light is near the band edge, the absorption of the semiconductor can be saturated at high light-intensity due to the nonlinear effect caused by band-filling. Semiconductor Q-switches have great flexibility since the bandgap can be engineered to fit specific laser wavelengths. However, damage to the semiconductor saturable absorber can be a problem due to the high intensity in Q-switched operation.
Solid-state saturable absorbers based on color centers were reported by several researchers [8-11]. Unfortunately, these saturable absorbers suffer from fading of the optical centers. Another approach to solid-state Q-switches is based on chromium doped in various crystalline hosts. In 1991, Andrauskas and Kennedy reported passive Q-switching of the flashlamp-pumped Nd:YAG and Nd:Glass lasers, as well as a diode-pumped Nd:YAG laser, with Cr:GSAG and Cr:GIGG solid-state saturable absorbers at room temperature [12]. Subsequently, several solid-state materials have been reported to work effectively as saturable absorber Q-switches for the solid-state lasers in the visible and near-infrared region in the past few years [13-29]. Specifically, Cr:YSO solid-state crystal was demonstrated to be an efficient passive Q-switch for the Cr:BeAl2O4 solid-state laser near 750 nm by Kuo et al. in 1995 [23].

The Cr:BeAl2O4 laser [30] is highly efficient and has important application in medical surgery (dermatology), water-vapor and temperature differential absorption LIDAR [31-36], solid-state laser pumping [37], and generation of ultraviolet laser radiation [38, 39]. Spectroscopic studies of the Cr:YSO and the observation of laser action from 77 up to 257 K were reported by Deka et al. in 1992 [40-42]. Room-temperature laser operation of the Cr:YSO was reported subsequently by Koetke et al. [43]. Besides laser application, Cr:YSO had also been demonstrated to be an efficient saturable absorber Q-switch for the ruby laser [44, 45], the Cr:LiSAF laser [46], the Cr:LiCAF laser [22], and the Cr:BeAl2O4 laser [23].

Theory of saturable absorber Q-switching has been extensively described in the literature [1, 2, 47-54]. In a previous work [22], we proposed a set of three coupled rate equations which may be utilized to model a four-level laser (for example, the widely used neodymium doped YAG, glass, and YVO4 lasers) or a three-level laser (for example, the ruby laser) that is passively Q-switched by a saturable absorber with excited state absorption. In Refs. [22] and [23], this theory was used to simulate the Cr:YSO Q-switched Cr:LiCAF and Cr:BeAl2O4 laser systems, respectively. The pulselength and the shape of the simulated Q-switched laser output was in good agreement with that observed experimentally. In Ref. [24], this theory was employed to investigate the behavior of ruby passive Q-switching with a Dy2+:CaF2 saturable absorber and to interpret the experiments with satisfactory results. The energy of the simulated Q-switched laser output was in good agreement with the result obtained experimentally.

In this paper, passive Q-switching performance of the Cr:BeAl2O4 laser with Cr:YSO solid-state saturable absorber is numerically investigated by solving the coupled rate equations with the Runge-Kutta-Fehlberg method. In the mean time, important factors such as the laser population inversion at various stages, the peak photon number inside the laser resonator, and the output energy and the pulselength of the Q-switched laser output are derived and utilized to theoretically evaluate the characteristics of Cr:BeAl2O4 passive Q-switching with Cr:YSO solid-state saturable absorber.

II. Passive Q-switching with slow-relaxing solid-state saturable absorbers

The Cr:YSO has an emission lifetime of 0.7 µs at room temperature [22], which is long compared to the duration of the Q-switched laser pulses. Therefore, Cr:YSO can be classified as a slow-relaxing saturable absorber. Following rate equations may be utilized to model a solid-state
laser passively Q-switched by a saturable absorber with excited state absorption [22]:

\[
\frac{dn}{dt} = [K_g N_g - K_a N_a - \beta K_a (N_a 0 - N_a) - \gamma_c] n, \tag{1}
\]

\[
\frac{dN_g}{dt} = R_p - \gamma_g N_g - \gamma K_g N_g n, \tag{2}
\]

\[
\frac{dN_a}{dt} = \gamma_a (N_a 0 - N_a) - K_a N_a n. \tag{3}
\]

The population reduction factor, \( \gamma \), equals to one for a four-level laser and two for a three-level laser. Other parameters used in these coupled rate equations are defined as following: \( n \) is the photon number inside the laser resonator; \( N_g \) is the population inversion of the laser; \( N_a \) is the ground state population of the saturable absorber; \( N_a 0 \) is the initial value of \( N_a \); \( \gamma_g = 1/\tau_g \) is the effective decay rate of the upper laser level; \( \gamma_a = 1/\tau_a \) is the effective saturable absorber relaxation rate, where \( \tau_a \) is the absorber emission lifetime; \( R_p \) is the pumping rate; \( \gamma_c = 1/\tau_c \) is the cavity decay rate, where \( \tau_c \) is the cavity lifetime; \( K_g = 2\sigma_g/\tau_g A_g \) is a coupling coefficient, where \( \sigma_g \) is the laser emission cross-section, \( \tau_g \) is the cavity round-trip transit time, and \( A_g \) is the effective laser beam area; \( K_a = 2\sigma_a/\tau_a A_a, \) where \( \sigma_a \) is the saturable absorber ground state absorption cross-section and \( A_a \) is the saturable absorber beam area; and \( \beta = \sigma_{ESA}/\sigma_a \) is the ratio of the excited state absorption cross-section, \( \sigma_{ESA} \), to the ground state absorption cross-section, \( \sigma_a \), of the saturable absorber.

Equations (1), (2) and (3) need to be solved numerically to obtain the behavior of a specific Q-switched laser system. However, important characteristics of a saturable absorber Q-switched laser system can be found from analyzing these three coupled rate equations. Following the method of Siegman [1], for passive Q-switching with a slow-relaxing saturable absorber, Eq. (1) becomes

\[
\frac{1}{n} \frac{dn}{dt} \cong \gamma_g 0 + (K_g 0^2 N_a 0 - K_g 2 N_g 0) \frac{n}{\gamma_g 0}, \tag{4}
\]

where

\[
\gamma_g 0 \equiv K_g N_g 0 - K_a N_a 0 - \gamma_c, \tag{5}
\]

and \( N_g 0 \) is the initial population inversion required for laser action.

When the photon number is small the photon number growth rate is dominated by the first term in the right hand side of Eq. (4), i.e. \( \gamma_g 0 \). Thus, before the saturable absorber starts to saturate the photon number inside the laser resonator increases at an initial growth rate \( \gamma_g 0 \). When the photon number is large the terms inside the parentheses start to take control. Therefore, for the passive Q-switching to occur both of these two terms have to be positive. After some manipulation, these two passive Q-switching criteria become

\[
K_g N_g 0 - K_a N_a 0 - \gamma_c > 0, \tag{6}
\]

\[
\frac{2\alpha_a L_a}{2\alpha_g L_g} \times \frac{\sigma_a}{\sigma_g} \times \frac{A_g}{A_a} > \gamma, \tag{7}
\]
where $\alpha_a$ and $L_a$ are the small signal absorption coefficient and crystal length of the saturable absorber, respectively, $\alpha_g$ and $L_g$ are the small signal emission coefficient and crystal length of the laser, respectively.

The physical meaning of Eq. (6) is that the gain medium has to be pumped to a level such that the gain of the laser is greater than the total loss of the laser cavity. Only under this circumstance can the laser signal be built up from the noise. Equation (6) can be regarded as the first threshold condition for passive Q-switching with slow-relaxing saturable absorbers. The physical meaning of Eq. (7) is that the saturable absorber must saturate first so that the net photon-number growth rate can turn upward which in turn allows the generation of a Q-switched laser pulse. If this condition is not satisfied, the photon number will start to decrease when the light intensity is high and a Q-switched laser pulse will never develop. Therefore, this equation can be regarded as the second threshold condition for passive Q-switching with slow-relaxing saturable absorbers. If the first term in Eq. (7) is close to unity, which is usually the case, and an internal lens is not used, i.e. $A_g \cong A_a$, then the absorption cross-section of the saturable absorber at the laser wavelength, $\sigma_a$, must be greater than the emission cross-section of the laser gain medium, $\sigma_g$, in order to effectively Q-switch the laser.

Since the build-up time of the Q-switched laser pulse is generally very short compared to pumping and relaxation times of the gain medium, it is reasonable to neglect pumping and spontaneous decay of the laser population inversion during pulse generation. With this assumption Eq. (2) becomes

$$\frac{dN_g}{dt} \cong -\gamma K_g N_g n. \quad (8)$$

When the light intensity is low almost all the population of the saturable absorber are in the ground state. Hence, the initial population inversion required for laser action can be approximated by setting the right hand side of Eq. (1) to zero and assuming $N_a \cong N_{a0}$, i.e.,

$$N_{g0} \cong K_a N_{a0} + \gamma_c \frac{N_{a0}}{K_g}. \quad (9)$$

When the light intensity is high most population in the ground state of the saturable absorber are promoted to the excited state. Therefore, the threshold population inversion after the bleaching of the saturable absorber can be derived by setting the right hand side of Eq. (1) to zero and assuming $N_a \cong 0$, i.e.,

$$N_{th} \cong \frac{\beta K_a N_{a0} + \gamma_c}{K_g}. \quad (10)$$

With Eq. (10) we can then rewrite Eq. (1) as:

$$\frac{dn}{dt} \cong K_g [N_g - N_{th}] n. \quad (11)$$

Equation (8) and Eq. (11) lead to following equation which relates $n$ and $N_g$:

$$n \cong \frac{1}{\gamma} \left[ N_{g0} - N_g - N_{th} \ln \left( \frac{N_{g0}}{N_g} \right) \right]. \quad (12)$$
As indicated in Eq. (11) the photon number reaches to a peak value $n_{\text{peak}}$ when $N_g$ is equivalent to $N_{\text{th}}$. Hence, from Eq. (12), we have

$$n_{\text{peak}} \approx \frac{1}{\gamma} \left[ N_{g0} - N_{\text{th}} - N_{\text{th}} \ln \left( \frac{N_{g0}}{N_{\text{th}}} \right) \right].$$

(13)

Or, equivalently, we may write

$$n_{\text{peak}} \approx \frac{N_{g0}}{\gamma} \left[ \frac{N_{g0}/N_{\text{th}} - 1 - \ln(N_{g0}/N_{\text{th}})}{N_{g0}/N_{\text{th}}} \right].$$

(14)

Equation (14) indicates that when $N_{g0}/N_{\text{th}}$ goes to infinity the peak photon number approaches to the maximum available population inversion $N_{g0}$ for a four-level laser system ($\gamma = 1$), and $N_{g0}/2$ for a three-level laser system ($\gamma = 2$).

After the release of the Q-switched laser pulse, the laser population inversion $N_g$ is depleted by the photon flux and reduces to a value below $N_{\text{th}}$. This final population inversion $N_f$ can be derived from Eq. (12) by setting $n \approx 0$ since the photon number is small after the release of the Q-switched laser pulse. Let $N_g = N_f$ and $n = 0$, then Eq. (12) becomes

$$N_{g0} - N_f - N_{\text{th}} \ln \left( \frac{N_{g0}}{N_f} \right) \approx 0.$$  

(15)

Equation (15) is transcendental and can be solved numerically. When $N_f$ is known, the output energy of the Q-switched pulse, accounting for the output coupling efficiency $\eta_c$, can be approximated by

$$E_{\text{out}} \approx \frac{N_{g0} - N_f}{\gamma} (h\nu)\eta_c.$$  

(16)

Using Eq. (13), we can approximate the peak power of the Q-switched laser output by

$$P_{\text{peak}} \approx \frac{n_{\text{peak}} h\nu}{\tau_c} \eta_c$$

$$\approx \frac{h\nu \eta_c}{\gamma \tau_c} \left[ N_{g0} - N_{\text{th}} - N_{\text{th}} \ln \left( \frac{N_{g0}}{N_{\text{th}}} \right) \right],$$

(17)

where $\tau_c$ is the cavity lifetime. The pulsewidth of the Q-switched laser pulse can then be approximated by

$$\tau_{\text{pulse}} \approx \frac{E_{\text{out}}}{P_{\text{peak}}}$$

$$\approx \frac{N_{g0} - N_f}{N_{g0} - N_{\text{th}} - N_{\text{th}} \ln \left( \frac{N_{g0}}{N_{\text{th}}} \right)} \tau_c.$$  

(18)
III. Performance of Cr:BeAl₂O₄ passive Q-switching with Cr:YSO saturable absorber

The laser rate equations (1) to (3) are numerically solved using the Runge-Kutta-Fehlberg method to investigate the performance of Cr:BeAl₂O₄ passive Q-switching with Cr:YSO solid-state saturable absorber. The results are shown in Figs. 1 and 2. The loss of the Q-switched laser system is defined from Eq. (1) as

\[ \text{Loss} \equiv \frac{K_a N_a + \beta K_a (N_{a0} - N_a)}{K_g} + \gamma_c. \]  

(19)

![Figure 1](image1.png)

**FIG. 1.** \(N_g, \text{Loss}, \) and \(n\) as functions of time.

![Figure 2](image2.png)

**FIG. 2.** \(N_g, \text{Loss}, \) and \(n\) near the occurrence of the first giant laser pulse as functions of time.
The parameters used in this simulation, obtained from published articles and experiments [22, 23, 30], are as following: laser wavelength = 750 nm, length of laser cavity = 30 cm, reflectivity of output coupler = 0.8, effective laser beam diameter = 2 mm, Cr:BeAl₂O₄ laser emission cross-section = $7.0 \times 10^{-21}$ cm², Cr:YSO ground-state absorption cross-section = $7.2 \times 10^{-19}$ cm², $K_g = 2.23 \times 10^{-10}$ sec⁻¹, $K_a = 2.29 \times 10^{-8}$ sec⁻¹, $\gamma_c = 1.0 \times 10^8$ sec⁻¹, $\gamma_g = 3.85 \times 10^3$ sec⁻¹, $\gamma_a = 1.43 \times 10^6$ sec⁻¹, $\beta = 0.33$, $R_p = 1.0 \times 10^{22}$ sec⁻¹, and $N_{a0} = 4.0 \times 10^{15}$, assuming no loss in the laser resonator except those due to the output coupling of the laser mirror and the absorption of the Cr:YSO saturable absorber.

Figure 1 shows $N_g$, $Loss$, and $n$ as functions of time. The results indicate that when the phonon number is low, $N_a$ is close to $N_{a0}$ and the loss of the laser system has an initial value of about $8.60 \times 10^{17}$, corresponding to $(K_a N_{a0} + \gamma_c)/K_g$ as indicated in Eq. (19). For the laser action to occur the laser has to be pumped, e.g. by the xenon flashlamp, so that the gain is greater than the loss, i.e. $N_g > Loss$. When this condition is satisfied the photon number starts to build up from the noise by depleting the laser population inversion and the Cr:YSO saturable absorber starts to saturate.

Figure 2 is an expanded picture of Fig. 1 near the occurrence of the first giant laser pulse. As shown in Fig. 2, when the photon number inside the laser resonator increases the loss decreases accordingly due to the bleaching effect of the Cr:YSO saturable absorber. According to Fig. 2, the loss reaches its minimum value early in the development of the Q-switched pulse. This is characteristic of an efficient saturable absorber Q-switch and is the result of a favorable cross-section ratio and a lifetime of the upper state of the saturable absorber which is much longer than the duration of the Q-switched pulse. The photon number reaches to the peak when the laser population inversion equals the cavity loss, i.e., when $N_g = Loss (= N_{th})$ $\cong 5.84 \times 10^{17}$. While the photon number reaches to the peak, the loss of the laser system reaches to its lowest value of $(\beta K_a N_{a0} + \gamma_c)/K_g$, as indicated in Eq. (19). Beyond this point the laser gain is smaller than the total loss of the laser system and the Q-switched laser pulse dies out quickly while the laser population inversion decreases gradually to a minimum value of about $3.50 \times 10^{17}$. The increase of the loss after the release of the Q-switched laser pulse is due to the relaxation of the saturable absorber population.

As shown in Fig. 1, it takes about 113 $\mu$s to develop the first giant laser pulse. The time spacing between subsequent Q-switched laser pulses is about 75 $\mu$s, which is smaller than that required for developing the first laser pulse because $N_g$ does not decrease to zero after the release of the first laser pulse. Note that the threshold population inversion after the bleaching of the saturable absorber calculated using Eq. (10), $5.84 \times 10^{17}$, is almost identical to that obtained in numerical simulation as indicated in Fig. 2. The initial laser population inversion required for laser action calculated using Eq. (9), $8.60 \times 10^{17}$, is about 6% lower than that observed in the simulation, $9.14 \times 10^{17}$. This is because we assume $N_g = Loss$ when deriving the equation while it is required that $N_g > Loss$ for the laser action to occur.

If we use the initial population inversion obtained in numerical simulation, the accuracy of evaluating the final laser population inversion after the release of the Q-switched laser pulse using Eq. (15) is within 2% when compared to the result of the numerical simulation ($3.45 \times 10^{17}$ vs. $3.50 \times 10^{17}$); and, the accuracy of evaluating the peak photon number using Eq. (13) is also within 4% when compared to the result of the numerical simulation ($6.84 \times 10^{16}$ vs. $6.60 \times 10^{16}$). The accuracies of using Eq. (16) and Eq. (18) to derive the output energy and the pulsewidth of the Q-switched laser pulse are within 1% ($116$ mJ vs. $115$ mJ) and 6% ($60.0$ ns vs. $56.8$ ns),
FIG. 3. Pulsewidth, peak photon number, and output energy as functions of $N_{a0}$ for several different $R$. 
The temporal profile of the giant laser pulse which contains an energy of 236 mJ and a pulsewidth of 26 ns.

respectively.

The pulsewidth (full width at half maximum), the peak photon number, and the energy of the laser output pulse (calculated by integrating the Q-switched laser pulse over a range covering the entire laser pulse) are solved numerically as functions of \( N_{a0} \) for several different reflectivities of output coupler, assuming other parameters remain unchanged. The results are shown in Fig. 3. A giant laser pulse with 236 mJ in energy and 26 ns in pulsewidth is obtained when \( N_{a0} = 1.0 \times 10^{16} \), \( R = 0.6 \), and \( R_p = 1.0 \times 10^{22} \). The temporal profile of this Q-switched laser pulse is shown in Fig. 4. It takes 364 \( \mu s \) to develop this first laser pulse and the time spacing between subsequent laser pulses is 279 \( \mu s \). The peak photon number inside the laser resonator is about \( 1.47 \times 10^{17} \) for this specific situation.

In general, when \( N_{a0} \) increases the pulsewidth decreases, and the peak photon number and the output energy of the Q-switched laser pulse increase for a specific reflectivity of output coupler. It indicates that better passive Q-switching performance, i.e., a shorter pulse with a higher output energy, can be obtained if a thicker saturable absorber or a saturable absorber of higher doping concentration is used.

The energy of the Q-switched Cr:BeAl\(_2\)O\(_4\) laser output pulse depends on the reflectivity of the output coupler since the output energy relates directly to the amount of output coupling. Higher output energy is usually obtained with an output coupler of a lower reflectivity. On the other hand, as shown in Fig. 3a, when \( N_{a0} \) is high the pulsewidth of the Q-switched laser output does not vary significantly when the output coupler is changed, except when the reflectivity of the output coupler is close to unity, since the change of output-coupler reflectivity causes only a small variation to the overall loss of the laser cavity which is dominated by the loss from the Cr:YSO saturable absorber Q-switch. Figures 3b and 3c show that a Q-switched laser pulse of a higher peak photon number and a lower output energy may be obtained with a higher reflectivity of the output coupler when \( N_{a0} \) is high. In fact, when \( N_{a0} \) is low, there exists an optimum reflectivity of output coupler with which best Q-switching performance may be obtained. This will become more clear in subsequent discussion.
FIG. 5. Photon number as a function of time for three different $N_{a0}$ with $R = 0.8$ and $R_p = 1.0 \times 10^{22}$.

FIG. 6. Temporal characteristics for the first two laser pulses as functions of $N_{a0}$ for several different $R$. 
FIG. 7. Pulse width, peak photon number, and output energy as functions of $R$ for several different $N_{a0}$. 
Figure 5 shows the photon number inside the laser resonator as a function of time for three different $N_{a0}$ with $R = 0.8$ and $R_p = 1.0 \times 10^{22}$. The temporal profile of the photon number has a narrower width and higher peak for a larger number of $N_{a0}$, as expected. Figure 6 shows the temporal characteristics for the occurrence of the first laser pulse and the duration between the first and second laser pulses as functions of $N_{a0}$ for several different reflectivities of the output coupler. As expected from theory, it takes more time to develop a Q-switched laser pulse with a higher $N_{a0}$ because it results in a higher loss due to absorption of the saturable absorber, and a lower reflectivity of the output coupler because it results in a higher loss due to larger amount of output coupling.

Figure 7 shows the pulsewidth, the peak photon number, and the energy of the laser output pulse as functions of $R$ for several different $N_{a0}$, assuming other parameters remain unchanged. Figure 7b indicates that the peak photon number increases when $R$ and $N_{a0}$ increase, as we have noticed in the previous discussion. On the other hand, in Figs. 7a and 7c we notice that the
FIG. 9. Pulsewidth, peak photon number, and output energy as functions of $R_p$ with $R = 0.8$ and $N_{a0} = 4 \times 10^{15}$.
pulsewidth and the output energy do not change markedly when $R$ varies except when $R$ is close to unity and $N_{a0}$ is lower than $3 \times 10^{15}$. As indicated in Fig. 7, there exists an optimum reflectivity of output coupler with which best Q-switching performance may be obtained when $N_{a0}$ is small; i.e., when the loss caused by the absorption of the saturable absorber is small compared to that caused by the output coupling.

Figure 8 shows the temporal characteristics for the occurrence of the first and the second laser pulses as functions of $R$ for several different $N_{a0}$. Similar to the results obtained in Fig. 6, we conclude that more time is required to develop the laser pulses for a low value of $R$ and a high value of $N_{a0}$.

Figure 9 shows the pulsewidth, the peak photon number, and the output energy as functions of the pumping rate, $R_p$, with $R = 0.8$ and $N_{a0} = 4 \times 10^{15}$. It is obvious that better Q-switching performance may be obtained with a higher pumping rate, as expected from theory. The temporal characteristics for the occurrence of the first two laser pulses as functions of $R_p$ are shown in Fig. 10. It takes less time to develop the laser pulses at higher pumping rates. When $R_p$ is lower than $1 \times 10^{22}$ sec$^{-1}$, the time required to develop the first pulse and the duration between the first two laser pulses increase rapidly. With $R_p = 3.0 \times 10^{22}$, a Q-switched laser pulse with 256 mJ in energy and 24 ns in pulsewidth is obtained when $N_{a0} = 1.0 \times 10^{16}$ and $R = 0.6$. It takes 79 $\mu$s to develop this first laser pulse and the time spacing between subsequent laser pulses is 54 $\mu$s. The peak photon number inside the laser resonator is about $1.75 \times 10^{17}$ for this specific situation.

In previous discussions we have assumed no loss in the laser resonator except those due to the output coupling of the laser mirror and the absorption of the Cr:YSGG saturable absorber. In real laser systems, dissipative cavity loss from scattering, diffraction, absorption, etc. may exist. Figure 11 shows the pulsewidth, the peak photon number, and the output energy as functions of dissipative cavity loss when $N_{a0} = 4.0 \times 10^{15}$, $R = 0.8$, and $R_p = 1.0 \times 10^{22}$. The pulsewidth has a value near 56 ns when dissipative cavity loss varies from 0 to as high as 0.5. However, the peak photon number and the output energy decrease markedly when dissipative cavity loss
FIG. 11. Pulseswidth, peak photon number, and output energy as functions of the dissipative cavity loss.
increases. Figure 12 shows the time required to develop the first laser pulse and the duration between the first two laser pulses as functions of dissipative cavity loss. It takes more time to develop the Q-switched laser pulses with a higher dissipative cavity loss, as expected.

IV. Conclusion

Theory of passive Q-switching with slow-relaxing solid-state saturable absorbers is investigated. First and second threshold conditions for passive Q-switching with slow-relaxing solid-state saturable absorbers are derived and discussed. Expressions of important factors such as the initial population inversion required for laser action, the threshold population inversion after the bleaching of the saturable absorber, the final population inversion after the release of the Q-switched laser pulse, and the peak photon number inside the laser resonator are derived and utilized to theoretically evaluate the characteristics of Cr:BeAl$_2$O$_4$ passive Q-switching with Cr:YSO solid-state saturable absorber.

Characteristics and performance of Cr:BeAl$_2$O$_4$ passive Q-switching with Cr:YSO solid-state saturable absorber are studied numerically using the Runge-Kutta-Fehlberg method for various Cr:YSO ground state absorption, output coupling, pumping rate, and dissipative loss situations. Better Q-switching performance, i.e., a shorter pulse with a higher output energy, can be obtained with a higher Cr:YSO ground state absorption, a higher pumping rate, and a lower dissipative loss inside the laser cavity. When the loss due to Cr:YSO ground state absorption is high compared to that caused by output coupling, more output energy may be obtained with a lower reflectivity of the output coupler. If the loss due to Cr:YSO ground state absorption is small compared to that caused by output coupling, there exists an optimum reflectivity of the output coupler with which best Q-switching performance may be obtained.

With typical optical pumping, output coupling, and Cr:YSO ground state absorption, an energy output of more than 200 mJ in a laser pulse duration of less than 30 ns may be obtained.
The results of the numerical simulation are in good agreement with theory proposed in this paper.

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